UNIVERSITY OF ILLINOIS
AT URBANA-CHAMPAIGN

Basic Error Analysis

Physics 401
Spring 2013
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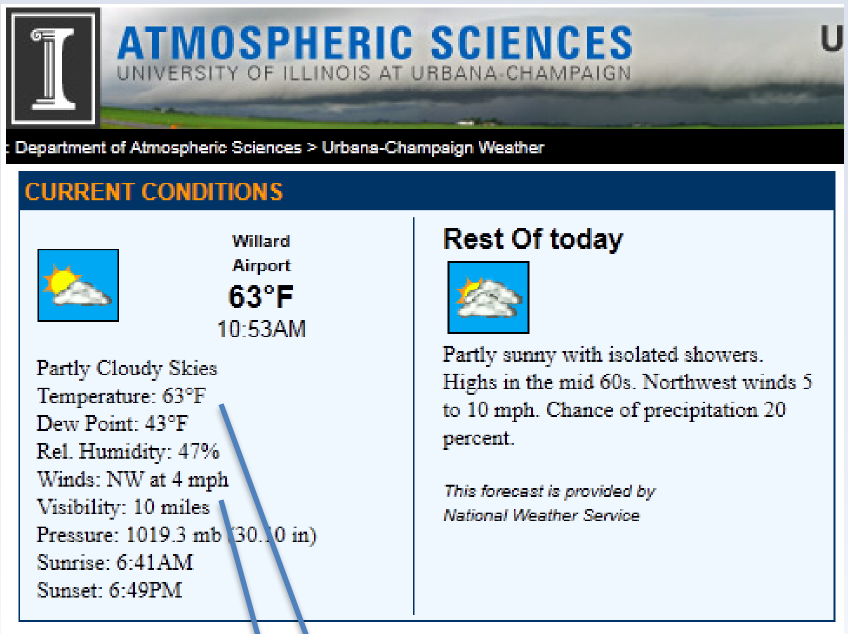
illinois.edu

Agenda

- Errors and uncertainties
- The Reading Error
- Accuracy and precession
- Systematic and statistical errors
- Fitting errors
- Appendix. Working with oil drop data

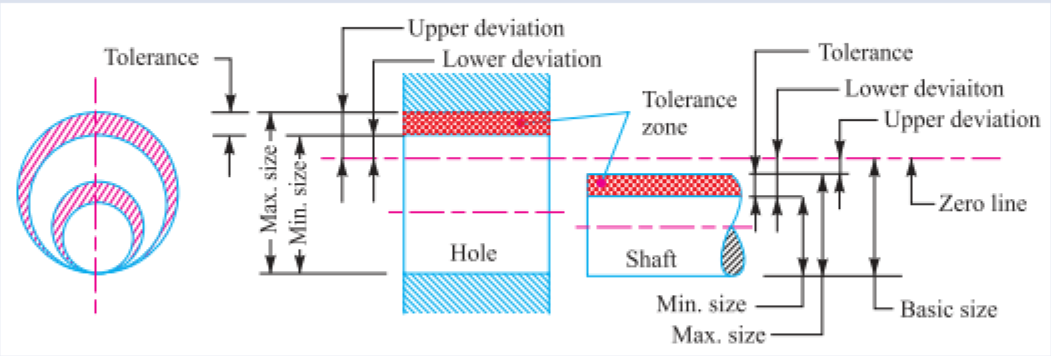
What and when we need to know about errors. Everyday life.



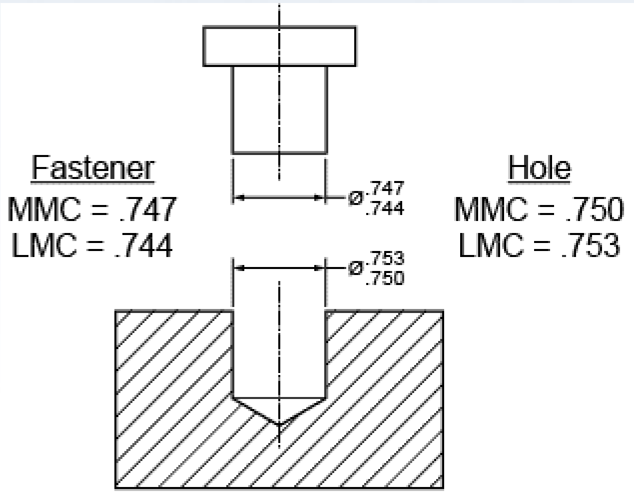
$T = 63°\text{F} \pm ?$ → Best guess $\Delta T \sim 0.5°\text{F}$

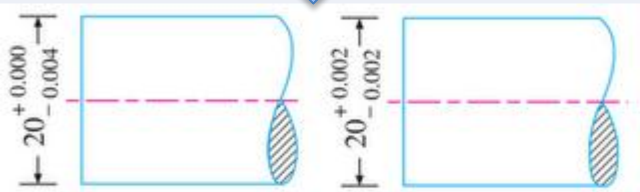
Wind speed 4mph$\pm$? → Best guess $\pm 0.5 mph$

What and when we need to know about errors. Industry.



Clearance fit





(a) Unilateral tolerance (b) Bilateral tolerance.

What and when we need to know about errors. Science.

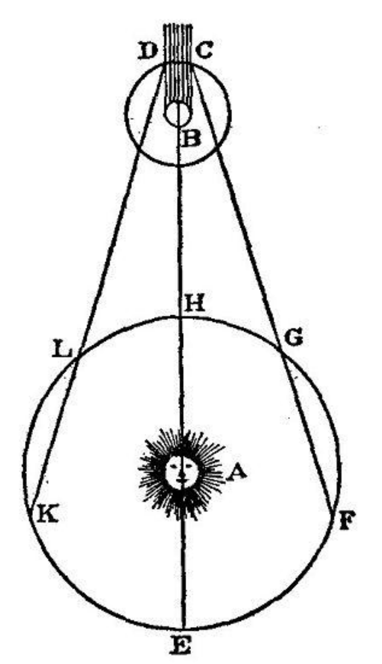


FIG. 70.

Measurement of the speed of the light

1675 Ole Roemer: 220,000 Km/sec

Ole Christensen Rømer
1644-1710

Does it make sense?
What is missing?

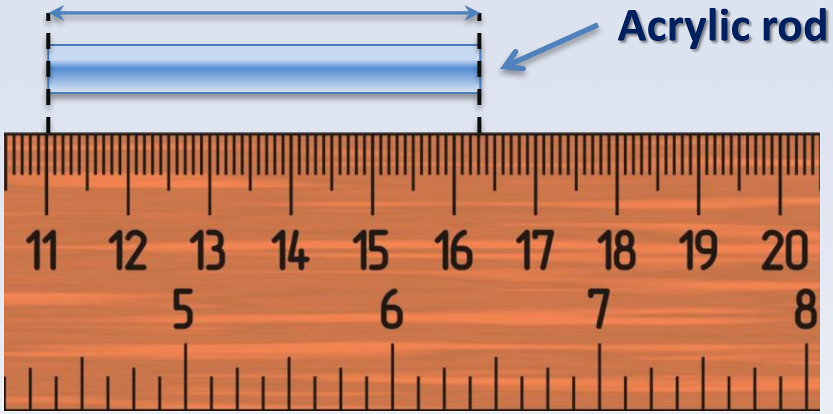
NIST Bolder Colorado c = 299,792,456.2±1.1 m/s.

Reading error

$\Delta L \cong 0.5mm$

L=53mm±ΔL(?)

Acrylic rod



$\Delta L \cong 0.03mm$

How far we have to go in reducing the reading error?

We do not care about accuracy better than 1mm

If ruler is not okay, we need to use digital caliper

Probably the natural limit of accuracy can be due to length uncertainty because of temperature expansion. For 53mm $\Delta L \cong 0.012mm/K$

Reading Error = $\pm\frac{1}{2}$ (least count or minimum gradation).

Reading error. Digital meters.

Fluke 8845A multimeter

Example Vdc (reading)=0.85V

$$\Delta V = 0.83 \times (1.8 \times 10^{-5}) + 1.0 \times (0.7 \times 10^{-5}) \cong 2.2 \times 10^{-5} = 22\mu V$$

8846A Accuracy

Accuracy is given as ± (% measurement + % of range)

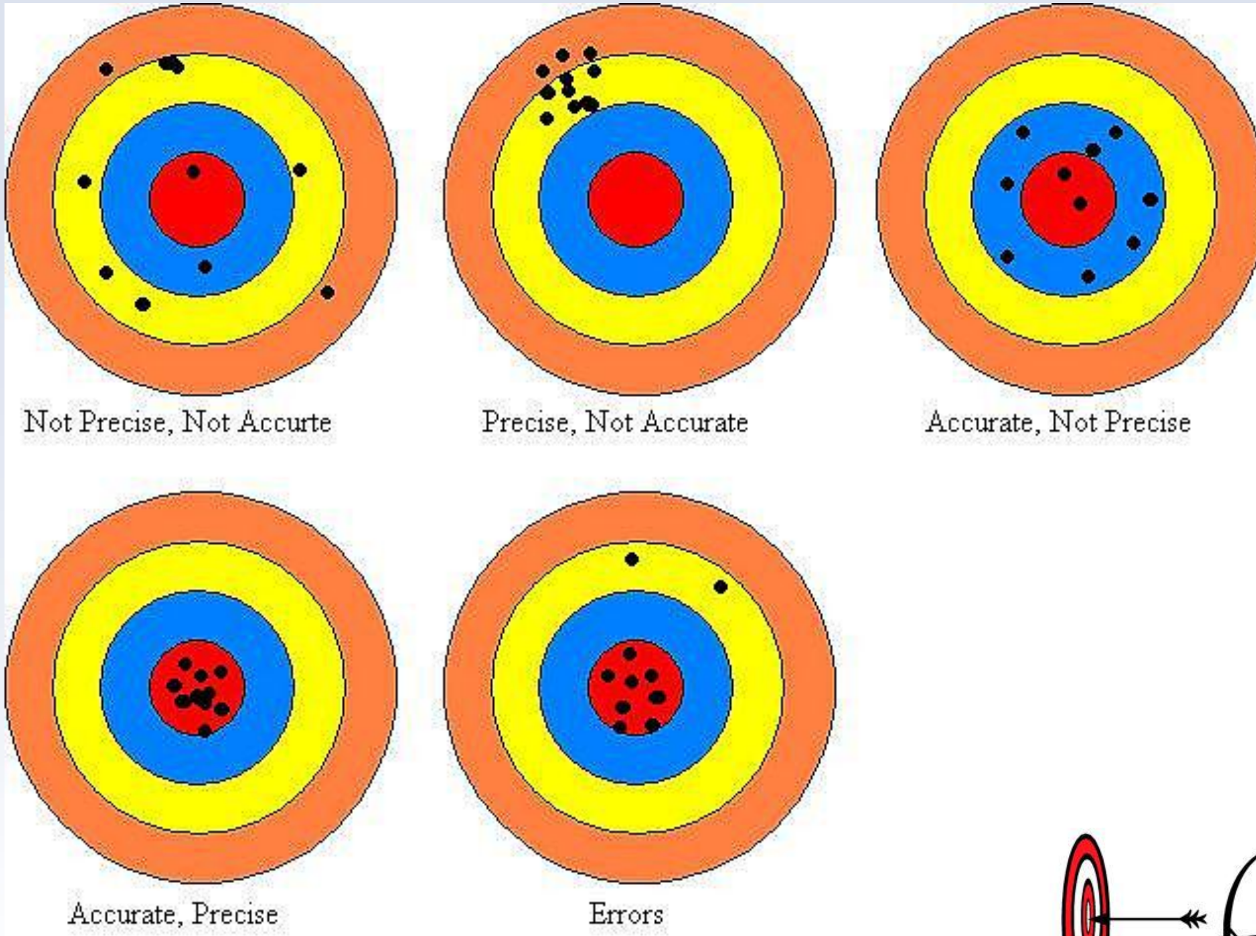
Range	24 Hour (23 ±1 °C)	90 Days (23 ±5 °C)	1 Year (23 ±5 °C)	Temperature Coefficient/ °C Outside 18 to 28 °C
100 mV	0.0025 + 0.003	0.0025 + 0.0035	0.0037 + 0.0035	0.0005 + 0.0005
1 V	0.0018 + 0.0006	0.0018 + 0.0007	0.0025 + 0.0007	0.0005 + 0.0001
10 V	0.0013 + 0.0004	0.0018 + 0.0005	0.0024 + 0.0005	0.0005 + 0.0001
100 V	0.0018 + 0.0006	0.0027 + 0.0006	0.0038 + 0.0006	0.0005 + 0.0001
1000 V	0.0018 + 0.0006	0.0031 + 0.001	0.0041 + 0.001	0.0005 + 0.0001

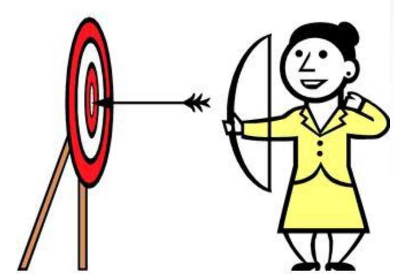
Accuracy and precession

The accuracy of an experiment is a measure of how close the result of the experiment comes to the true value

Precision refers to how closely individual measurements agree with each other

Accuracy and precession





Systematic and random errors

- ***Systematic Error:*** reproducible inaccuracy introduced by faulty equipment, calibration or technique.

- ***Random errors***: Indefiniteness of results due to finite precision of experiment. Measure of fluctuation in result after repeatable experimentation.

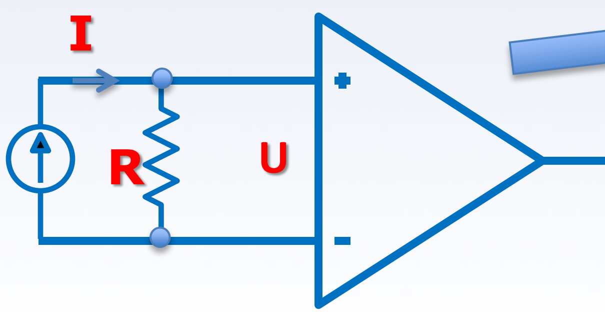
Philip R. Bevington "Data Reduction and Error Analysis for the Physical sciences", McGraw-Hill, 1969

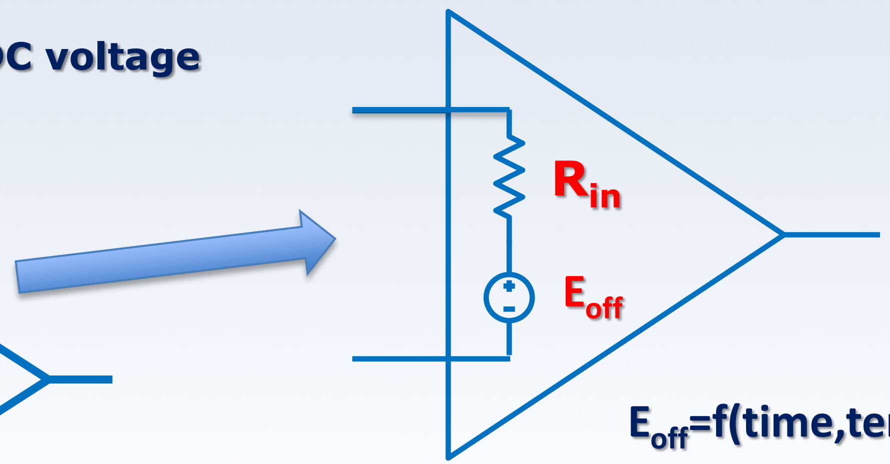
Systematic errors

Sources of systematic errors: **poor calibration of the equipment, changes of environmental conditions, imperfect method of observation, drift and some offset in readings etc.**

Example #1: measuring of the DC voltage

Current source





E_{off}=f(time,temperature)

expectation

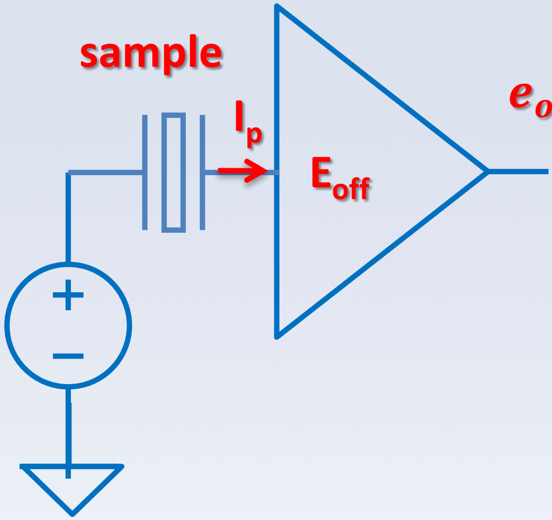
U=R*I

actual result

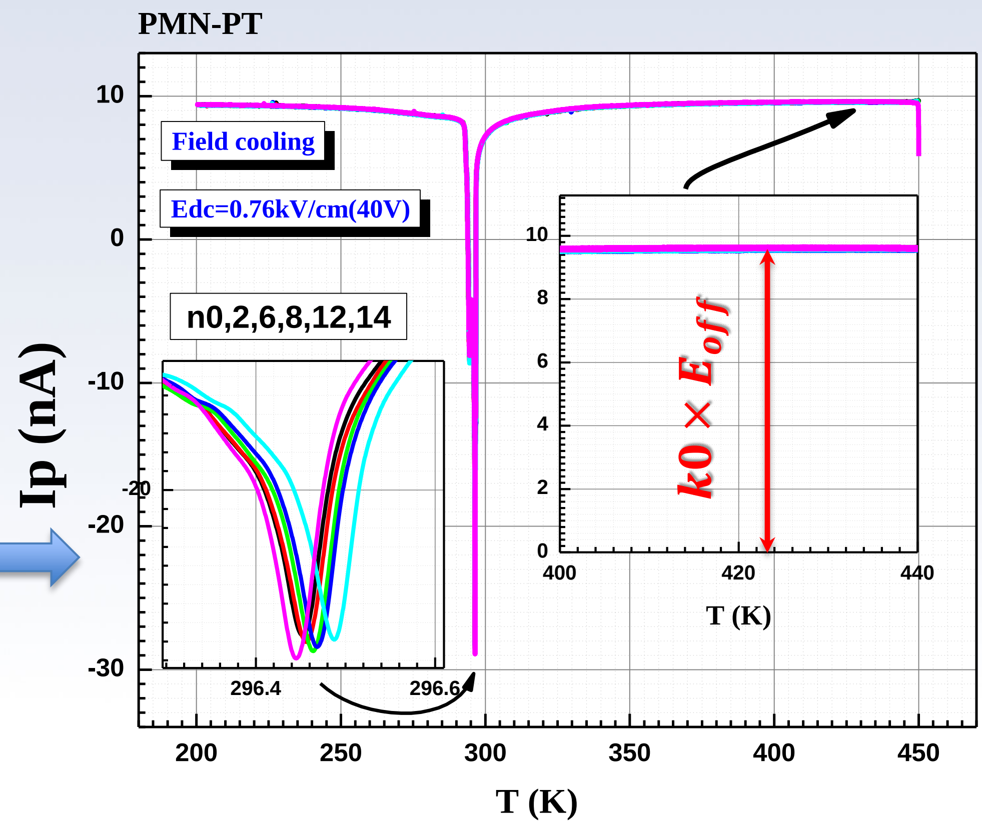
$$U = \frac{R * I - \left(\frac{R}{R_{in}}\right) E_{off}}{\left(1 + \frac{R}{R_{in}}\right)}$$

Systematic errors. Example.

Example #2: measuring of the polarization current

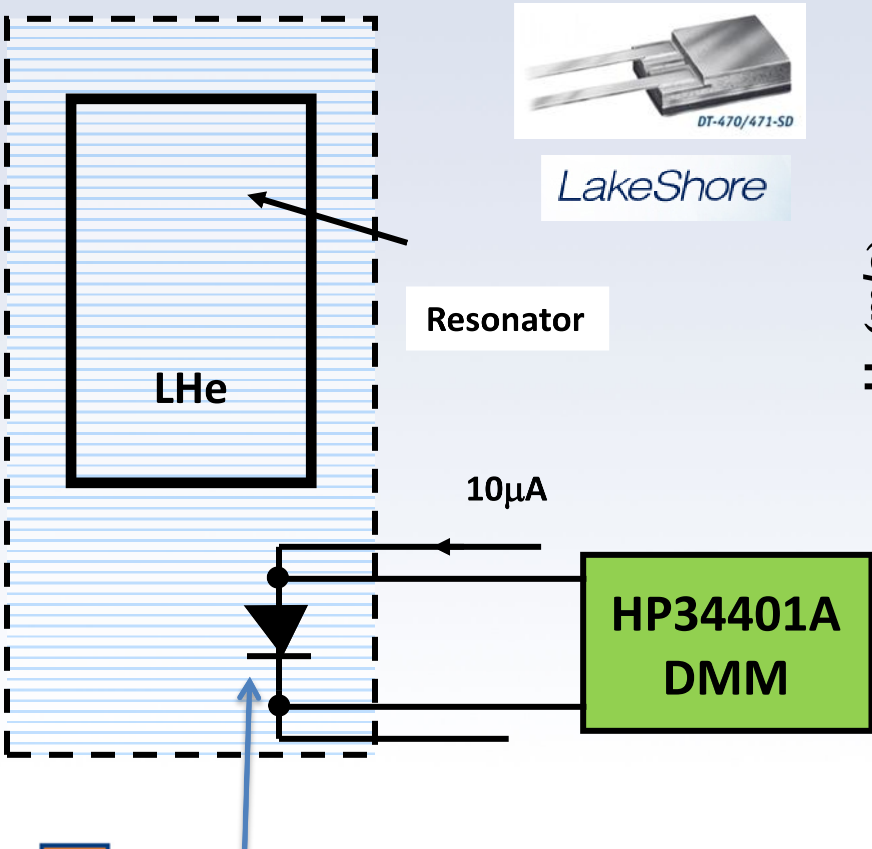


$$e_{out} = k1 \times I_p + k0 \times E_{off}$$



Systematic errors

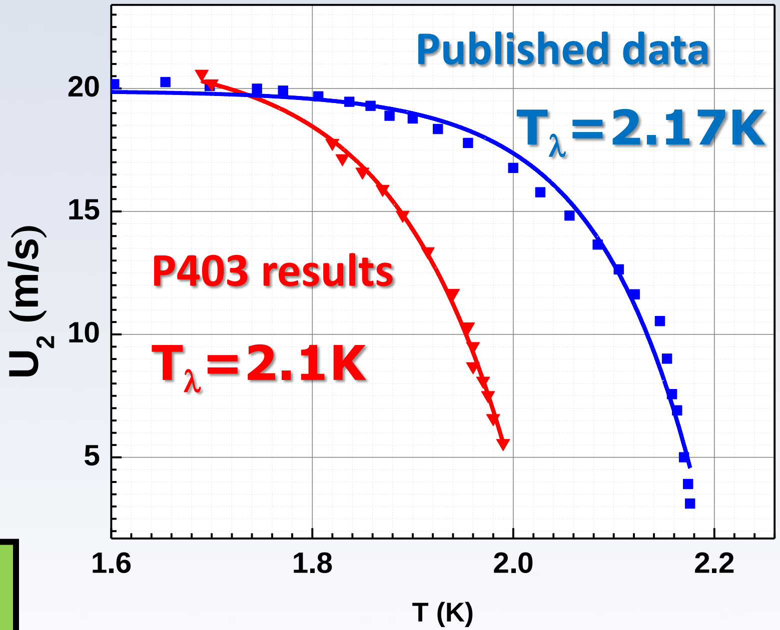
Example #3: poor calibration

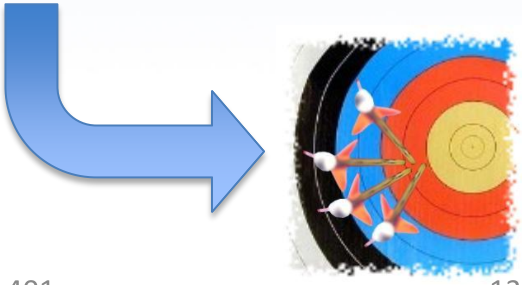


LakeShore

Temperature sensor

Measuring of the speed of the second sound in superfluid He4





Random errors

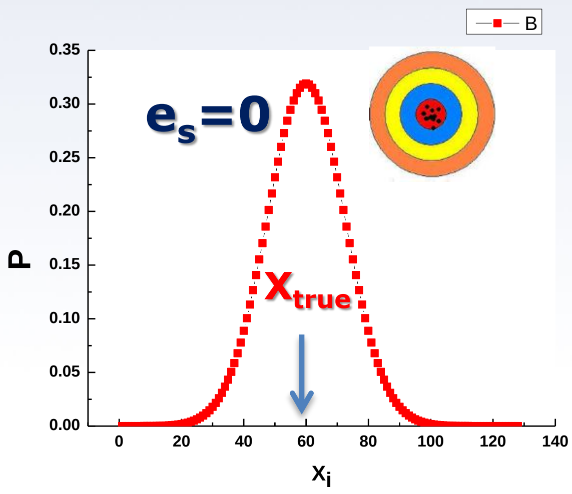
Result of measurement

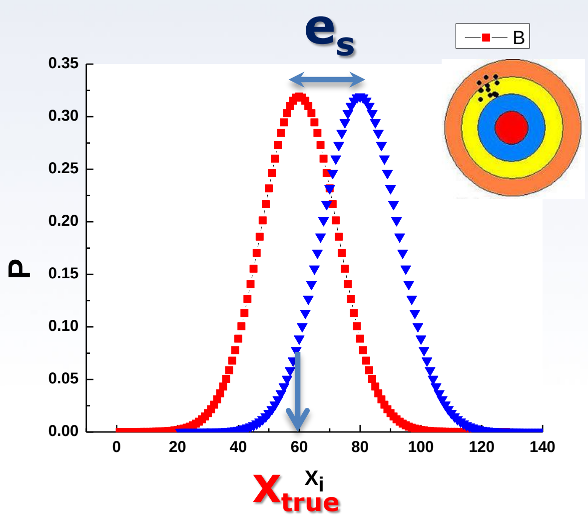
Systematic error

$$X_{meas} = X_{true} + e_s + e_r$$

Correct value

Random error





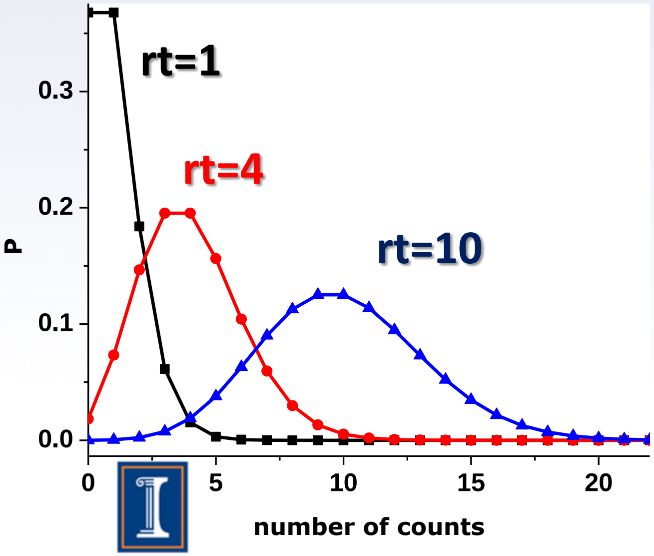
Random errors. Poisson distribution

Siméon Denis Poisson
(1781-1840)

$$P_n(t) = \frac{(rt)^n}{n!} e^{-rt} \quad n = 0,1,2,\ldots$$

r: decay rate [counts/s] t: time interval [s]

➔ $P_n(rt)$: Probability to have n decays in time interval t



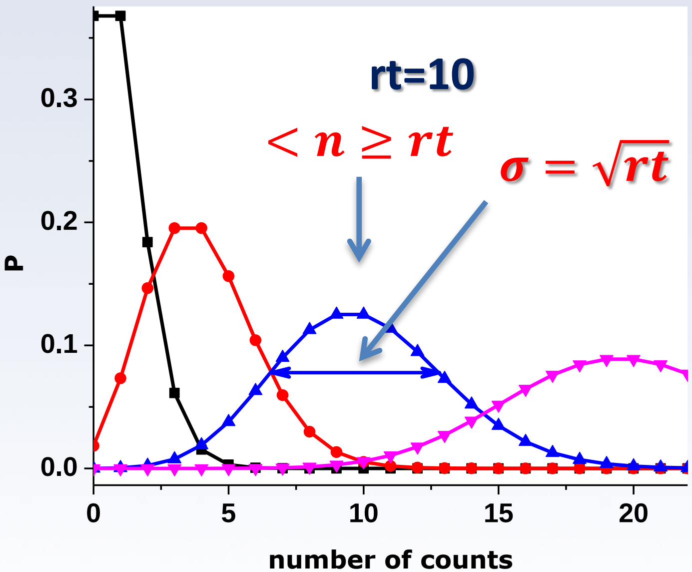
A statistical process is described through a Poisson Distribution if:

- **random process** → for a given nucleus probability for a decay to occur is the same in each time interval.
- **universal probability** → the probability to decay in a given time interval is same for all nuclei.
- **no correlation between two instances** (the decay of on nucleus does not change the probability for a second nucleus to decay.

Poisson distribution

$$P_n(t) = \frac{(rt)^n}{n!} e^{-rt} \quad n = 0,1,2,\ldots$$

r: decay rate [counts/s] ***t***: time interval [s]
➔ ***$P_n(rt)$*** : Probability to have n decays in time interval **t**



Properties of the Poisson distribution:

$$\sum_{n=0}^{\infty} P_n(rt) = 1$$, **probabilities sum to 1**

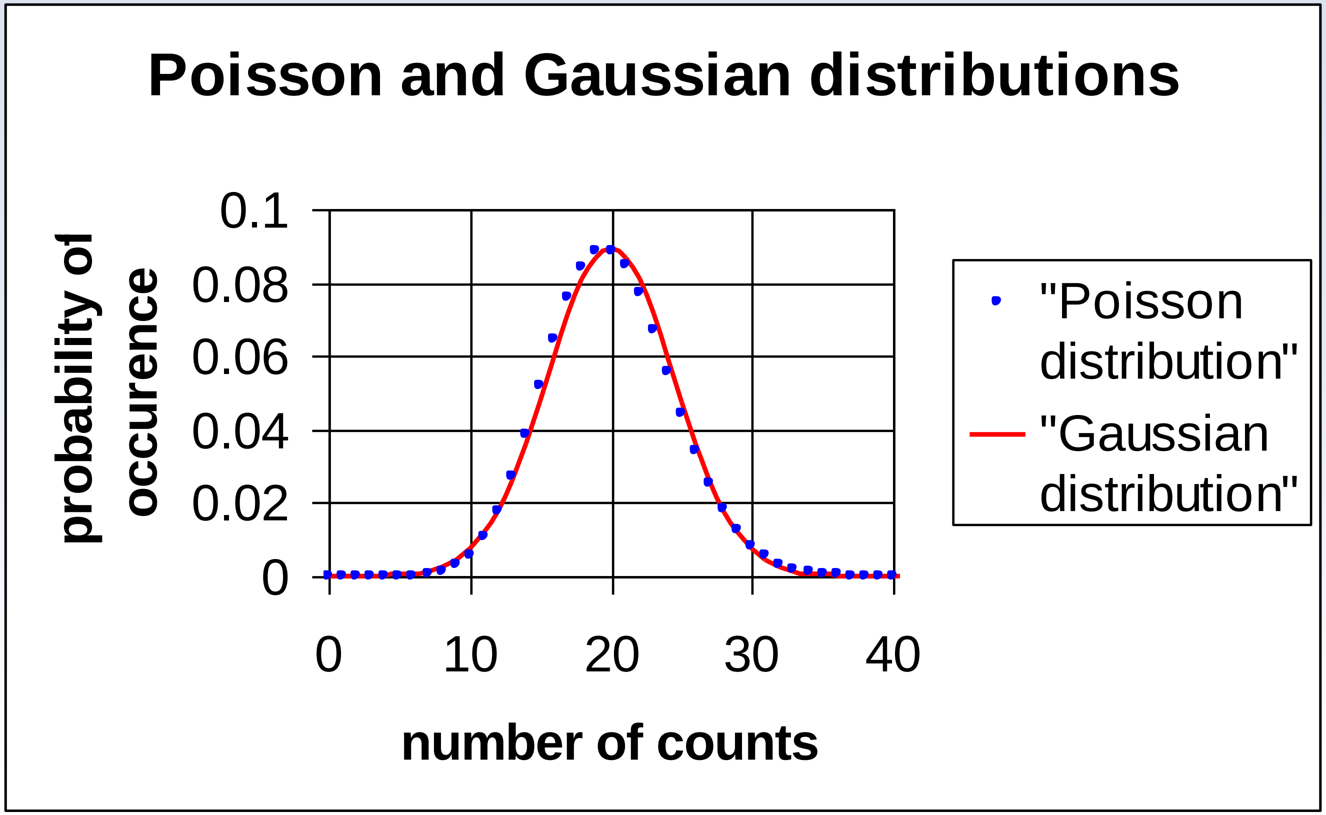
$$<n> = \sum_{n=0}^{\infty} n \cdot P_n(rt) = rt$$, **the mean**

$$\sigma = \sqrt{\sum_{n=0}^{\infty}(n - <n>)^2 P_n(rt)} = \sqrt{rt}$$,

standard deviation

Poisson distribution at large *rt*

$$P_n(t) = \frac{(rt)^n}{n!} e^{-rt} \quad n = 0,1,2,\ldots$$

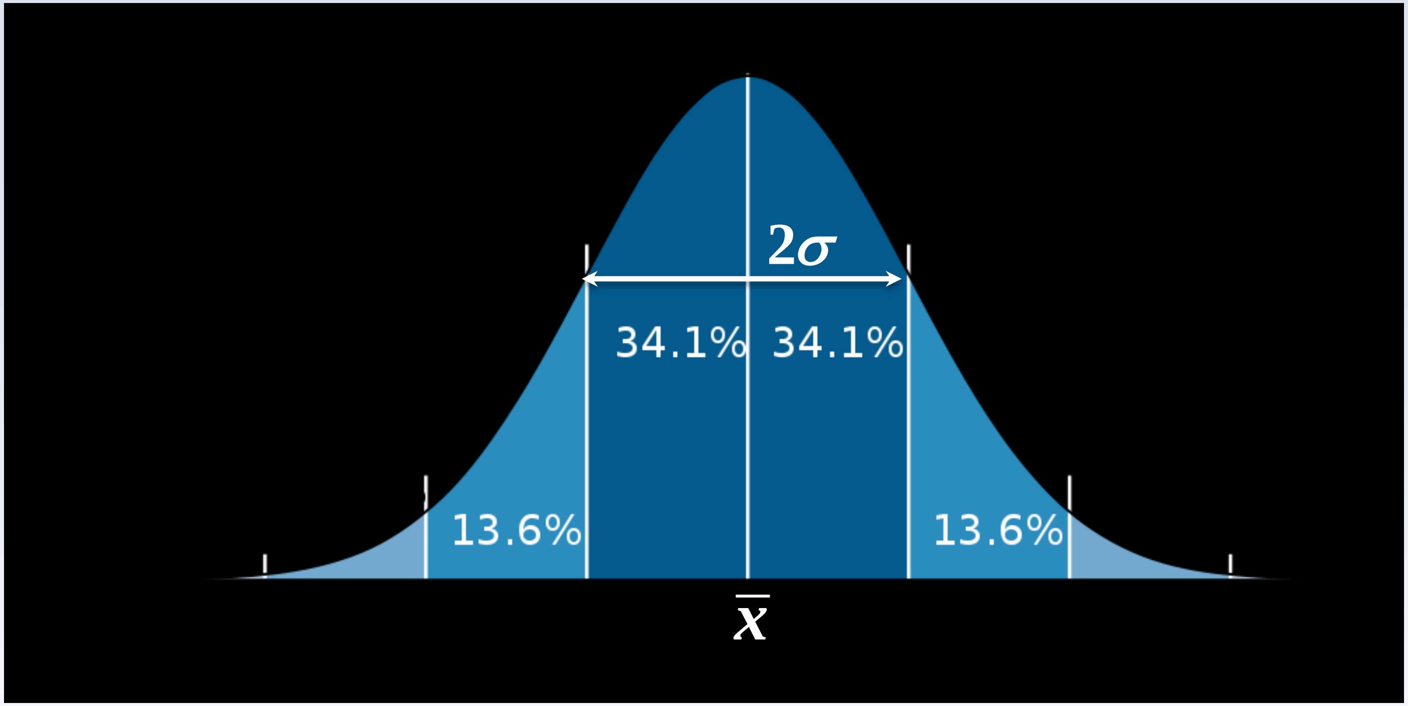


Carl Friedrich Gauss (1777–1855)

$$P_n(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\bar{x})^2}{2\sigma^2}}$$

Gaussian distribution: continuous

Normal (Gaussian) distribution

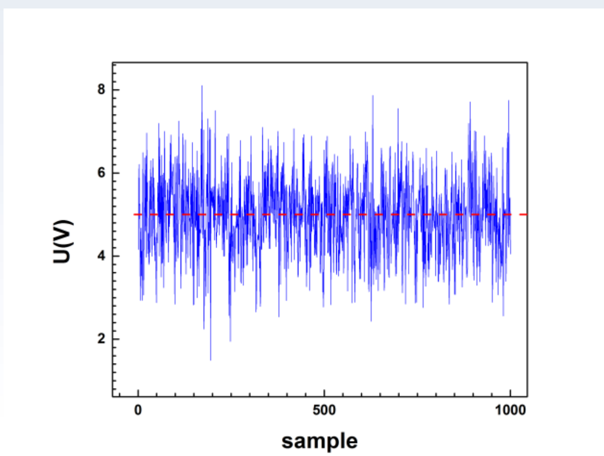


$$P_n(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\bar{x})^2}{2\sigma^2}}$$

Error in the mean is given as $\frac{\sigma}{\sqrt{N}}$

Measurement in presence of noise

Source of noisy signal

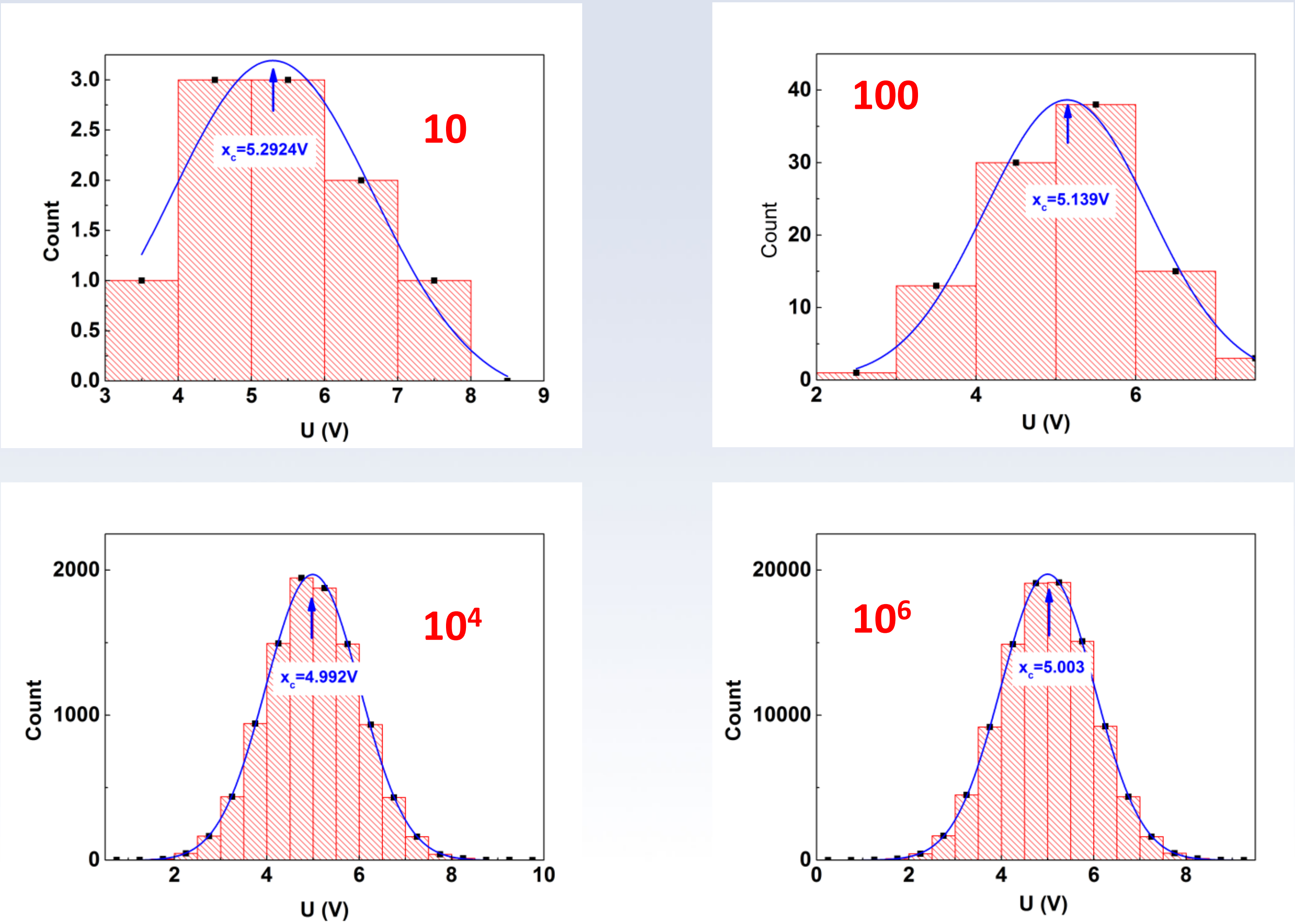


4.89855
5.25111
2.93382
4.31753
4.67903
3.52626
4.12001
2.93411

Expected value 5V

Actual measured values

Measurement in presence of noise



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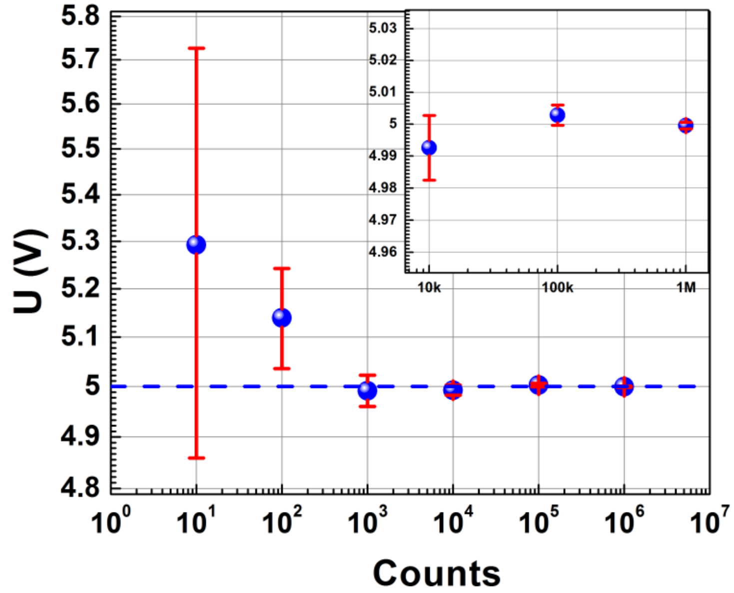
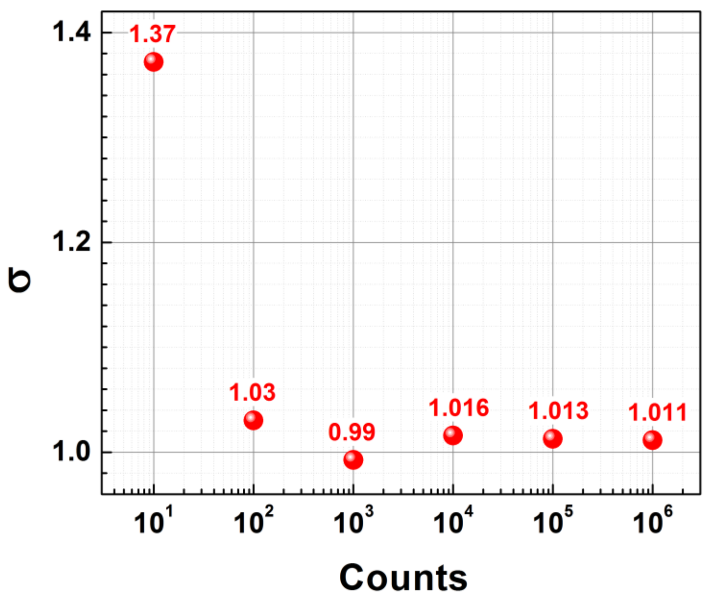
Measurement in presence of noise





Result → $U = x_c \pm \frac{\sigma}{\sqrt{N}}$

σ - standard deviation
N – number of samples

For $N=10^6$ U=4.999±0.001 0.02% accuracy

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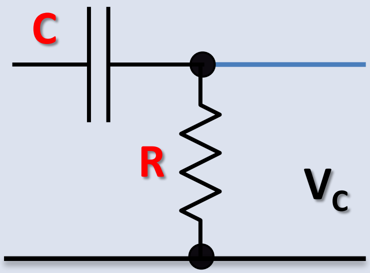
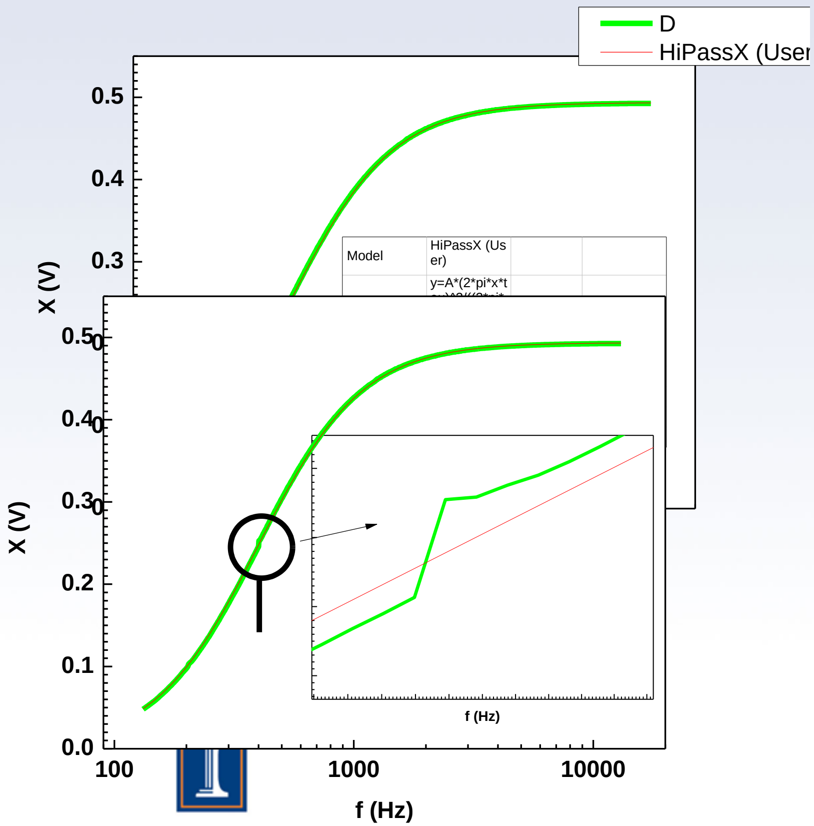
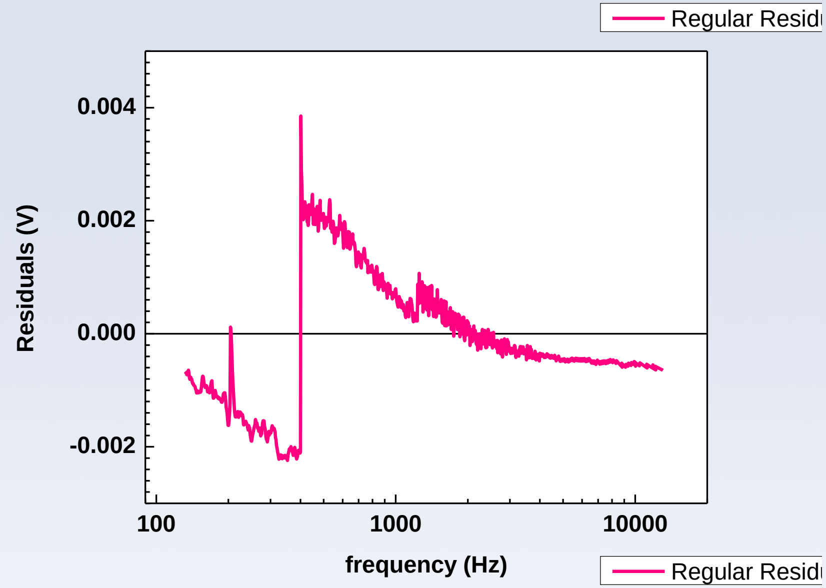
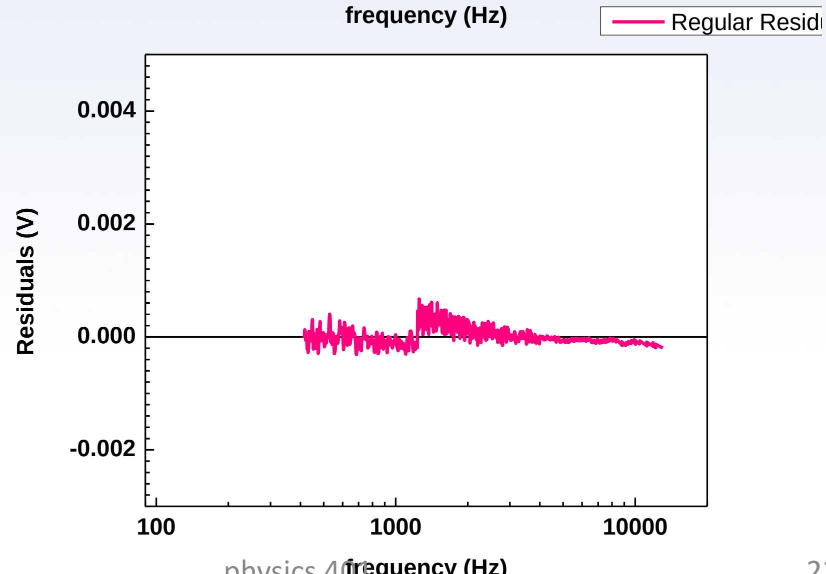
Fitting errors



Hi pass filter

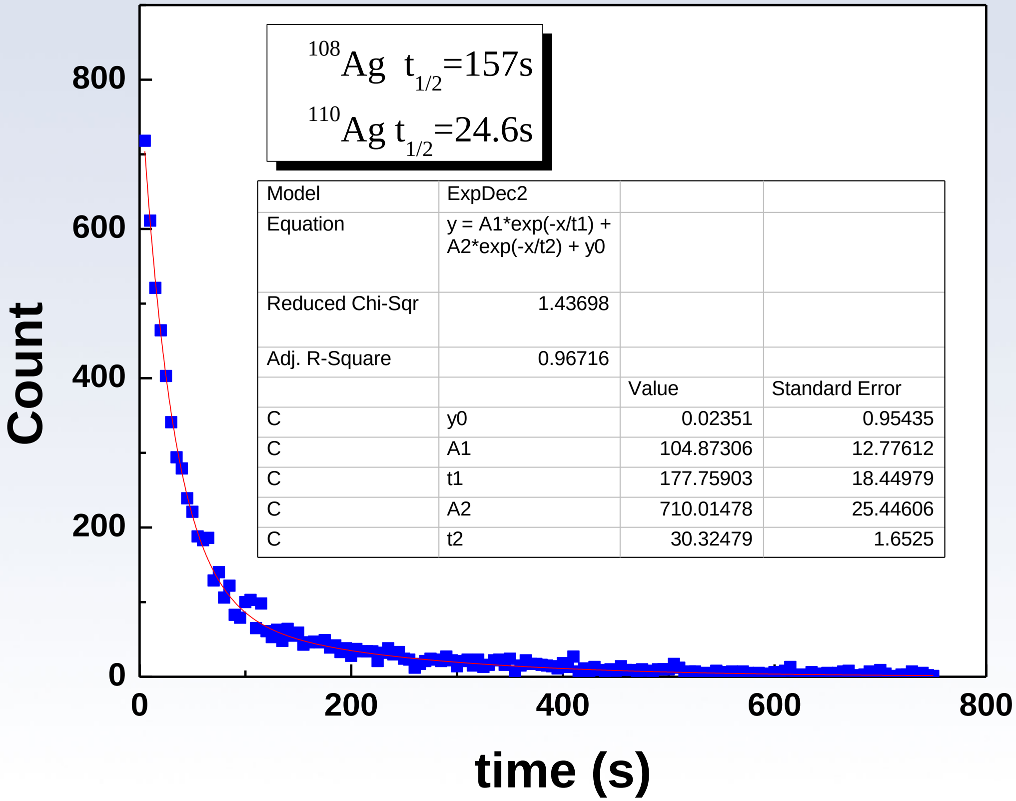




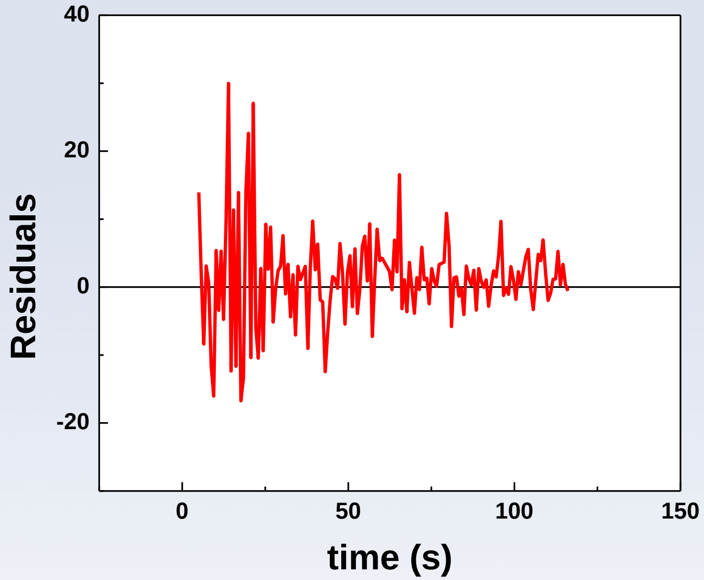


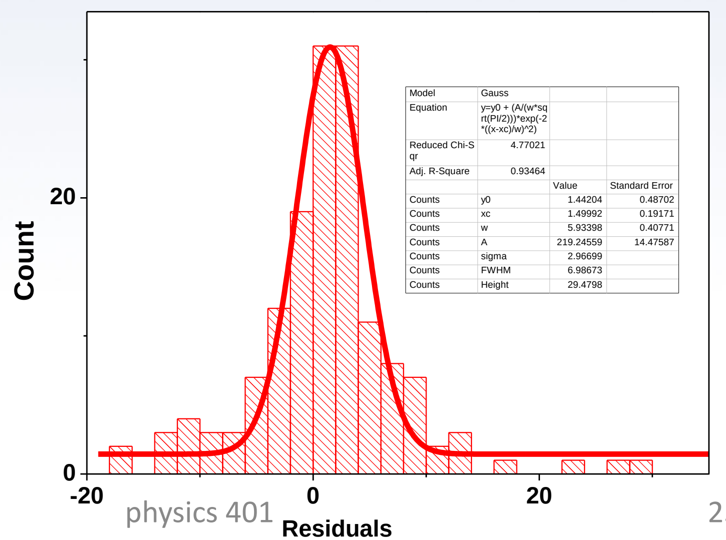
Fitting errors

Ag β decay



Model	ExpDec2		
Equation	y = A1*exp(-x/t1) + A2*exp(-x/t2) + y0		
Reduced Chi-Sqr	1.43698		
Adj. R-Square	0.96716		
		Value	Standard Error
C	y0	0.02351	0.95435
C	A1	104.87306	12.77612
C	t1	177.75903	18.44979
C	A2	710.01478	25.44606
C	t2	30.32479	1.6525





Model	Gauss		
Equation	y=y0 + (A/(w*sqrt(PI/2)))*exp(-2*((x-xc)/w)^2)		
Reduced Chi-Sqr	4.77021		
Adj. R-Square	0.93464		
		Value	Standard Error
Counts	y0	1.44204	0.48702
Counts	xc	1.49992	0.19171
Counts	w	5.93398	0.40771
Counts	A	219.24559	14.47587
Counts	sigma	2.96699	
Counts	FWHM	6.98673	
Counts	Height	29.4798	

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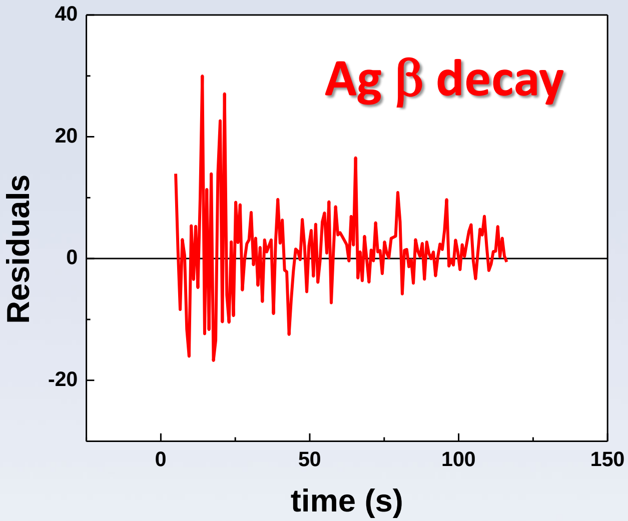
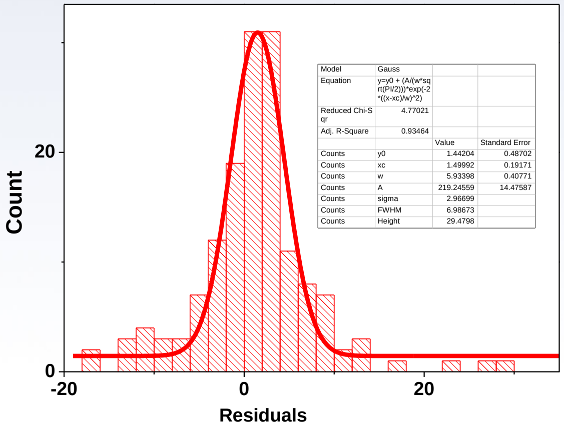
Fitting. Analysis of the residuals





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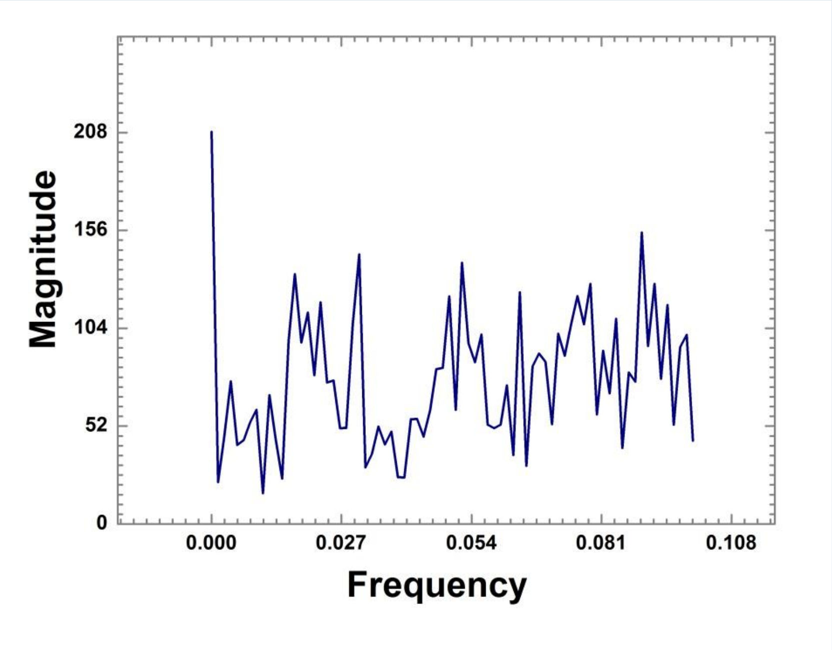
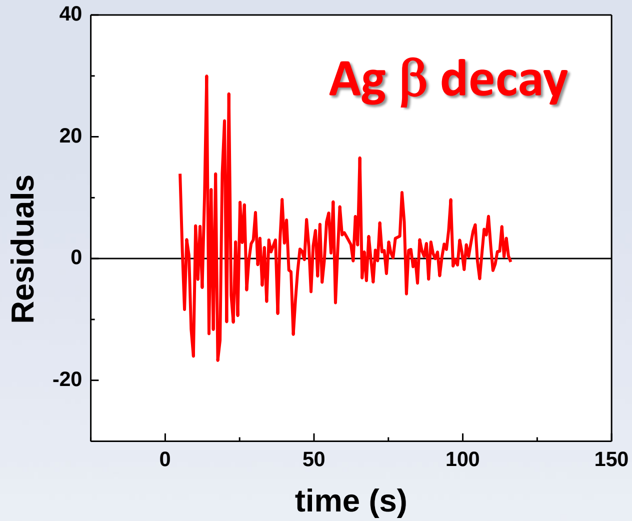
Test 1. Fourier analysis

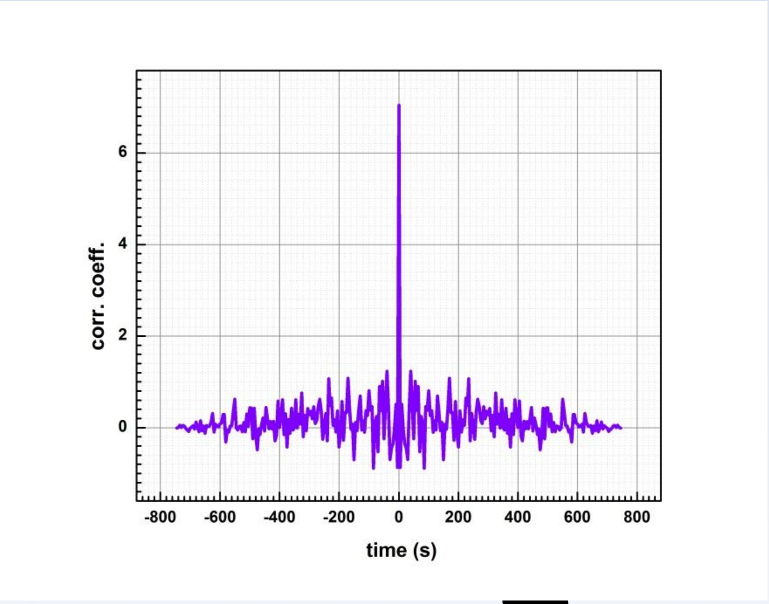


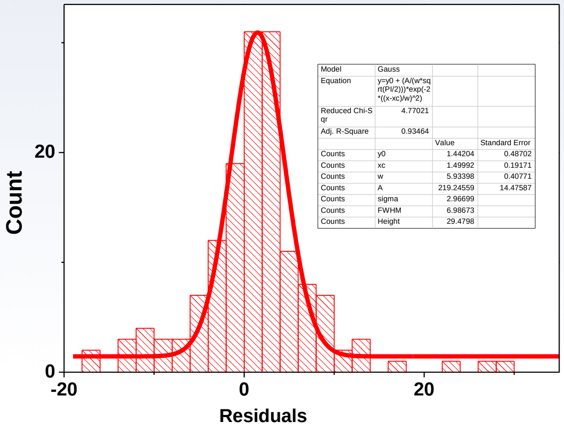
No pronounced frequencies found

Fitting. Analysis of the residuals



Test 1. Autocorrelation function





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Counts	A	219.24559	14.47587
Counts	sigma	2.96699	
Counts	FWHM	6.98673	
Counts	Height	29.4798	

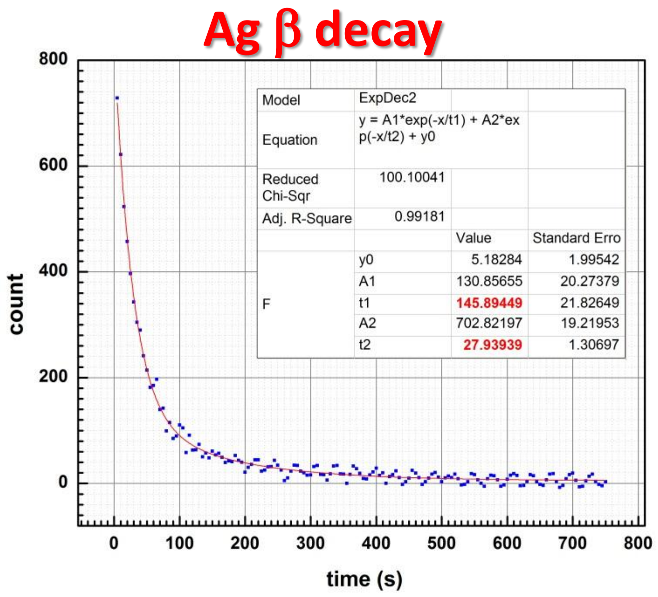
Correlation function

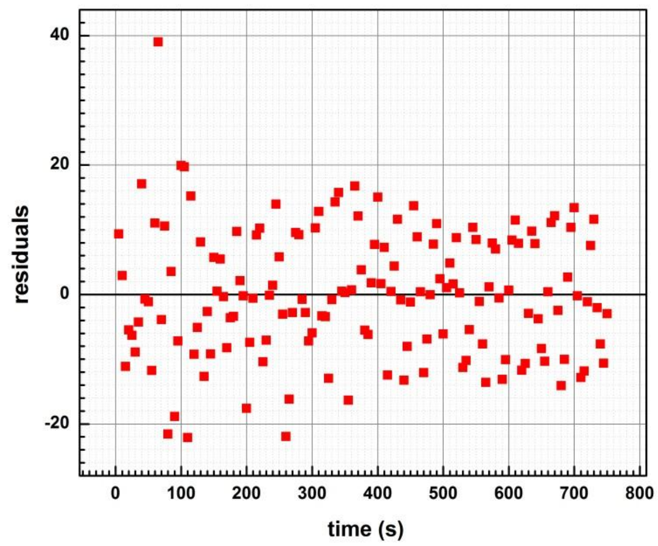
$$y(m) = \sum_{n=0} f(n)g(n-m)$$

autocorrelation function

$$y(m) = \sum_{n=0}^{M-1} f(n)f(n-m)$$

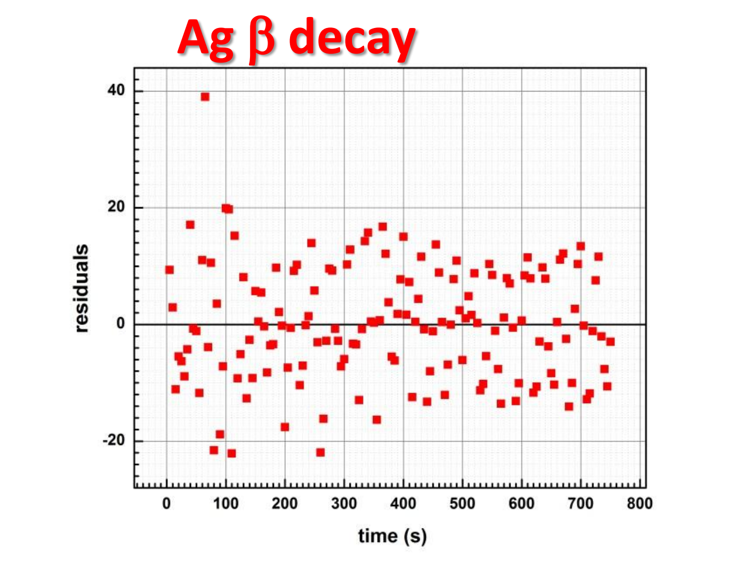
Fitting. Analysis of the residuals. Non "ideal" case

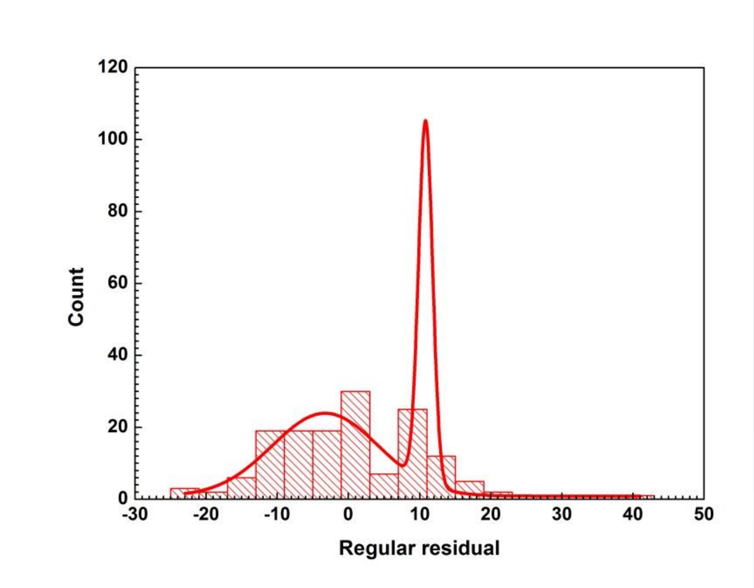




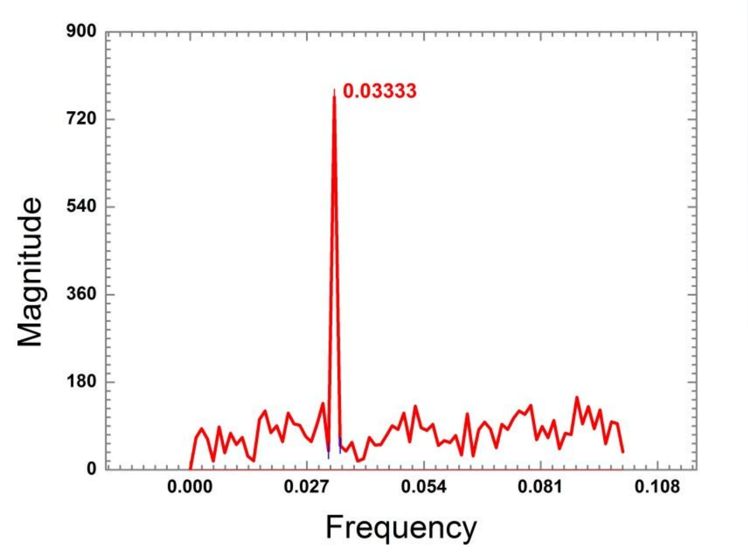
	Clear experiment	Data + "noise"
t_1(s)	177.76	145.89
t_2(s)	30.32	27.94

Fitting. Analysis of the residuals. Non "ideal" case

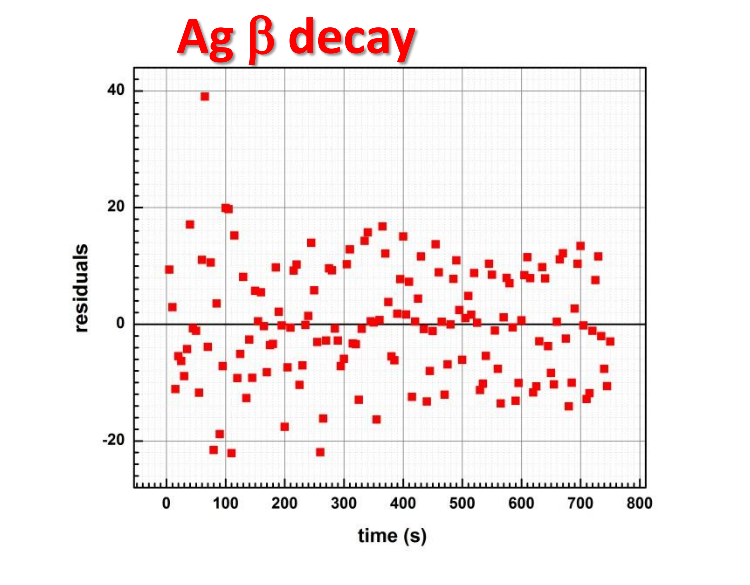


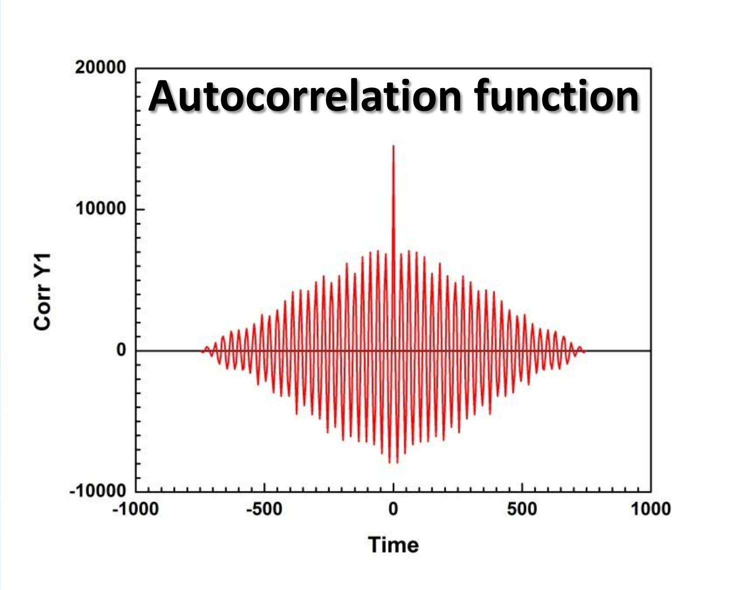


Histogram does not follow the normal distribution and there is frequency of 0.333 is present in spectrum



Fitting. Analysis of the residuals. Non "ideal" case

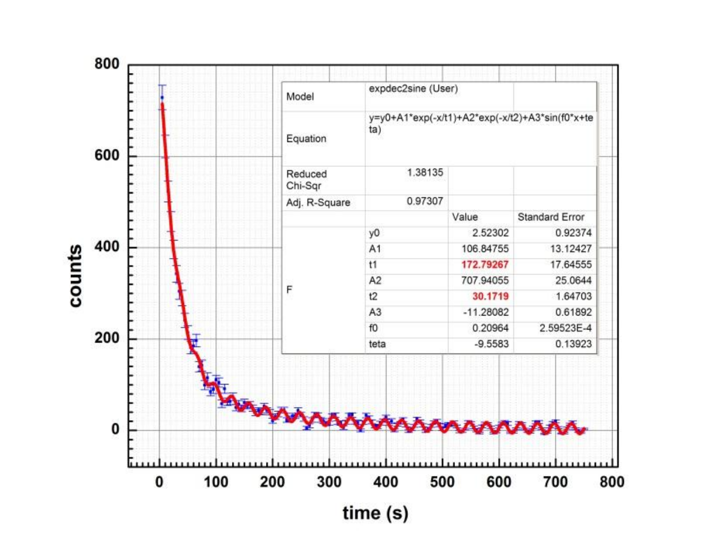


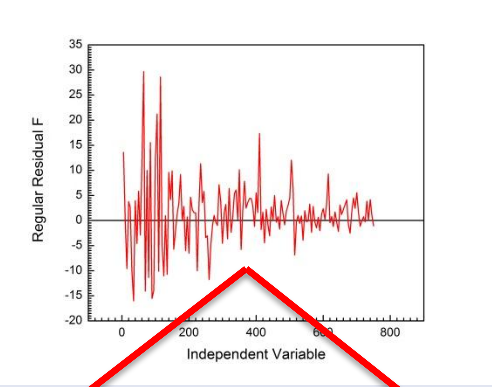


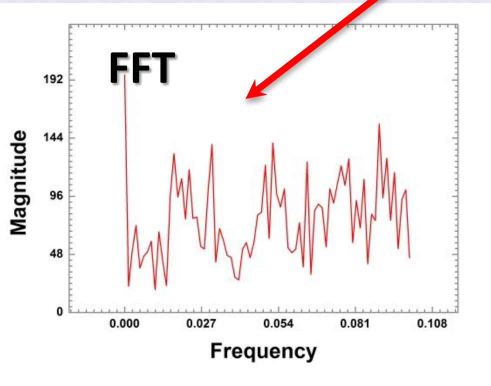
Conclusion: fitting function should be modified by adding an additional term:

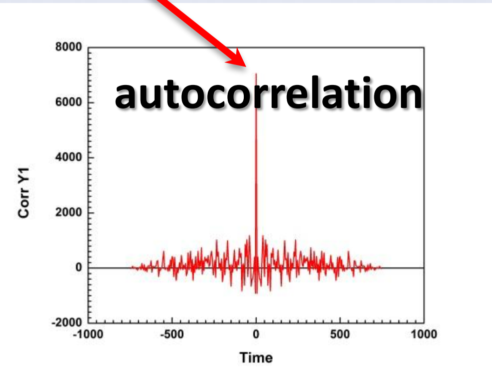
$$y(t) = y_0 + A_1 \exp\left(\frac{-t}{t_1}\right) + A_2 \exp\left(\frac{-t}{t_2}\right) + A_3 \sin(\omega t + \theta)$$

Fitting. Analysis of the residuals. Non "ideal" case







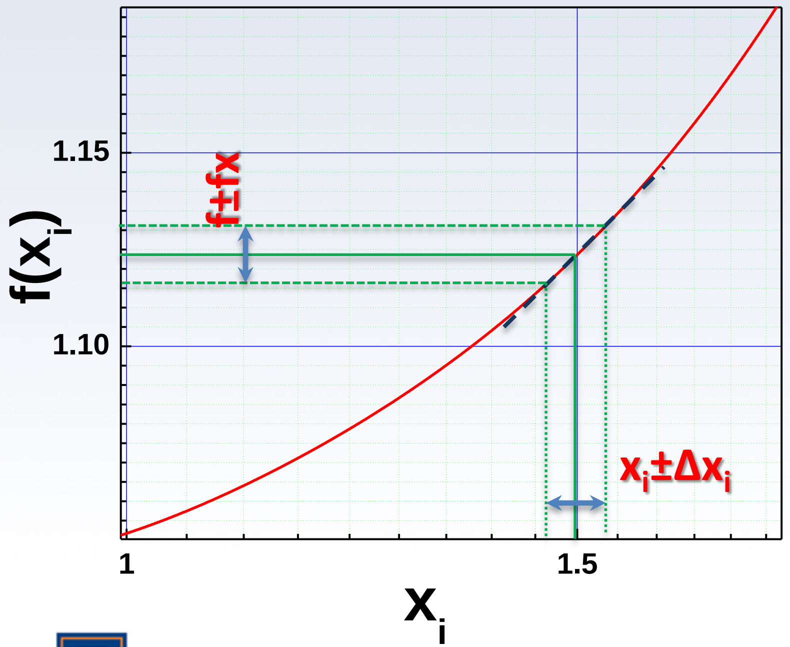


	Clear experiment	Data + noise	Modified fitting
t_1(s)	177.76	145.89	172.79
t_2(s)	30.32	27.94	30.17

Error propagation

y = f(x1, x2 ... xn)

$$\Delta f(x_i, \Delta x_i) = \sqrt{\sum_{i=1}^{n} \left[\frac{\partial f}{\partial x_i} \right]^2 \cdot \Delta x_i^2}$$



Error propagation. Example.

Derive resonance frequency *f* from measured inductance *L±ΔL* and capacitance *C±ΔC*

$$f(L,C)=\frac{1}{2\pi}\sqrt{\frac{1}{LC}}$$

$$L_1=10\pm1\mathrm{mH},\quad C_1=10\pm2\mu\mathrm{F}$$

$$\Delta f(L,C,\Delta L,\Delta C)=\sqrt{\left[\frac{\partial f}{\partial L}\right]^2\cdot\Delta L^2+\left[\frac{\partial f}{\partial C}\right]^2\Delta C^2}$$

$$\frac{\partial f}{\partial L}=\frac{-1}{4\pi}C^{-\frac{1}{2}}L^{-\frac{3}{2}};$$

$$\frac{\partial f}{\partial C}=\frac{-1}{4\pi}L^{-\frac{1}{2}}C^{-\frac{3}{2}}$$

Results:

$f(L_1,C_1)$=503.29212104487Hz

Δf=56.26977Hz

$f(L_1,C_1)$=503.3±56.3Hz

Error propagation. Millikan oil drop experiment.

$$Q = F \bullet S \bullet T = \left(\frac{1}{f_c^{3/2}}\right)\frac{9\pi d}{V}\sqrt{\frac{2\eta^3 x^3}{g\rho}}\frac{1}{\sqrt{t_g}}\left(\frac{1}{t_g}+\frac{1}{t_{rise}}\right)$$

$$\Delta Q = \sqrt{(S \bullet T)^2 \Delta F^2 + (F \bullet T)^2 \Delta S^2 + (F \bullet S)^2 \Delta T^2}$$

$$T = \frac{1}{\sqrt{t_g}}\left(\frac{1}{t_g}+\frac{1}{t_{rise}}\right)$$

$$\Delta T = \sqrt{\left(\frac{3/2}{t_g^{5/2}}+\frac{1/2}{t_g^{3/2}}\frac{1}{t_{rise}}\right)^2 \Delta t_g^2 + \left(\frac{1}{t_g^{1/2}}\frac{1}{t_{rise}^2}\right)^2 \Delta t_{rise}^2}$$

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Appendix. Analyzing of the statistical data.

Step 1. Collect your data + parameters of the experiment in:

\\Phyaplportal\PHYCS401\Common\Origin templates\Oil drop experiment\Section L1.opj

Use different columns for each student or team. This Origin project is for data collecting only but not for data analysis. For data analysis you have to copy these data and experiment parameters obtained by different students/team and paste it in one in your personal Origin project.

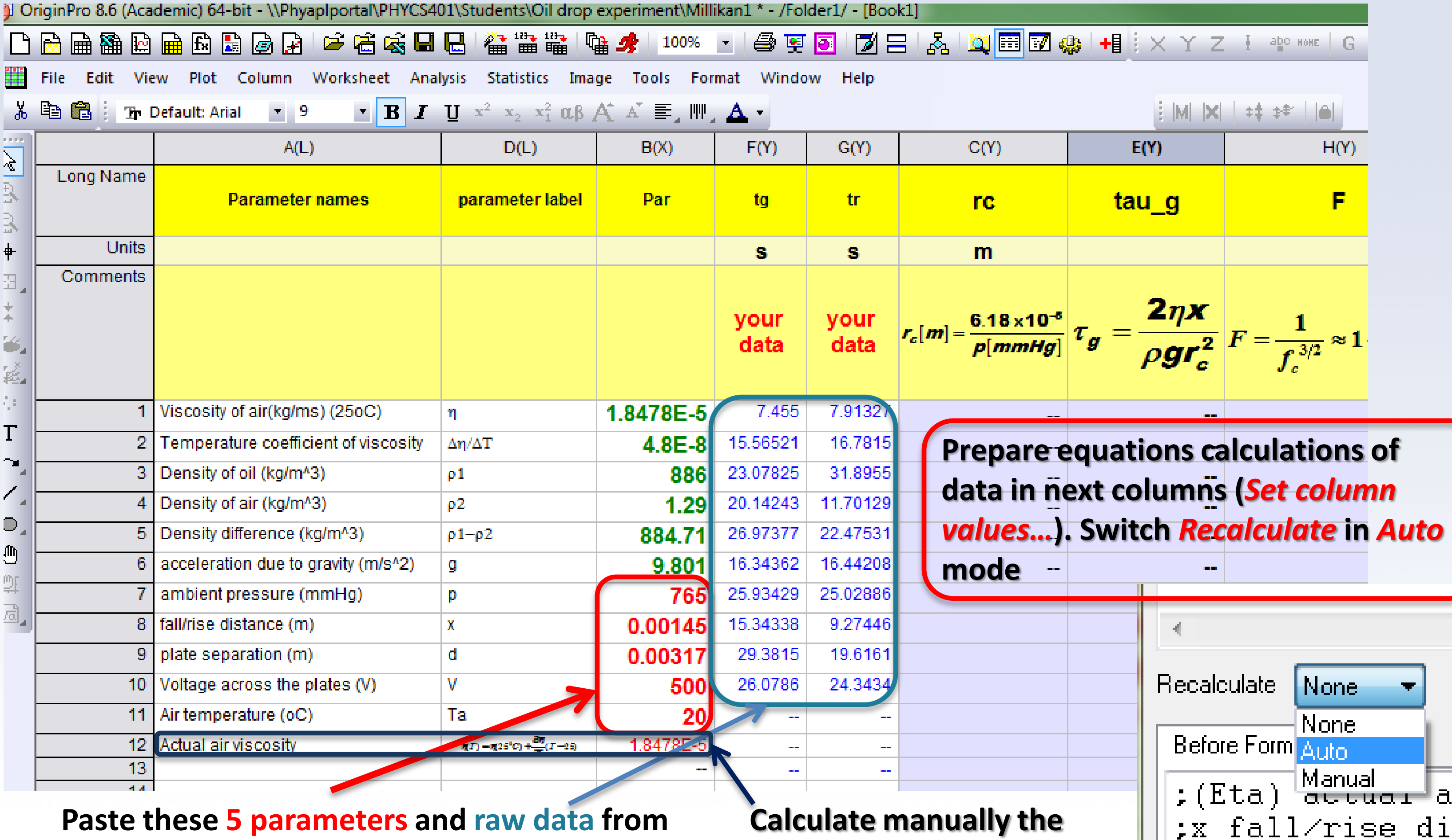
	A(L)	B(Y)	C(Y)	D(Y)
Long Name	parameter label	Par	tg	tr
Units				
Comments	student1, student2	student1, student2	student1, student2	student1, student2
1	p	765	15.56521	16.7815
2	x	0.00145	23.07825	31.8955
3	d	0.00317	20.14243	11.70129
4	V	500	26.97377	22.47531
5	Ta	20	16.34362	16.44208
6			25.93429	25.02886
7			15.34338	9.27446
8			29.3815	19.6161
9			26.0786	24.3434

Setup and environmental parameters (column B) | **Raw data** (columns C, D)

Appendix. Analyzing of the statistical data.

Step 2. Working on personal Origin project

Make a copy of the Millikan1 project to your personal folder and open it



	A(L)	D(L)	B(X)	F(Y)	G(Y)	C(Y)	E(Y)	H(Y)
Long Name	Parameter names	parameter label	Par	tg	tr	rc	tau_g	F
Units				s	s	m		
Comments				your data	your data	$r_c[m]=\frac{6.18\times10^{-8}}{p[mmHg]}$	$\tau_g=\frac{2\eta x}{\rho g r_c^2}$	$F=\frac{1}{f_c^{3/2}}\approx 1$
1	Viscosity of air(kg/ms) (25oC)	η	1.8478E-5	7.455	7.91327	--	--	
2	Temperature coefficient of viscosity	Δη/ΔT	4.8E-8	15.56521	16.7815	--	--	
3	Density of oil (kg/m^3)	ρ1	886	23.07825	31.8955	--	--	
4	Density of air (kg/m^3)	ρ2	1.29	20.14243	11.70129	--	--	
5	Density difference (kg/m^3)	ρ1−ρ2	884.71	26.97377	22.47531			
6	acceleration due to gravity (m/s^2)	g	9.801	16.34362	16.44208	--	--	
7	ambient pressure (mmHg)	p	765	25.93429	25.02886			
8	fall/rise distance (m)	x	0.00145	15.34338	9.27446			
9	plate separation (m)	d	0.00317	29.3815	19.6161			
10	Voltage across the plates (V)	V	500	26.0786	24.3434			
11	Air temperature (oC)	Ta	20	--	--			
12	Actual air viscosity	$\eta(T)=\eta(25^\circ C)+\frac{\partial\eta}{\partial T}(T-25)$	1.8478E-5	--	--			
13			--	--	--			

Prepare equations calculations of data in next columns (*Set column values...*). Switch *Recalculate* in *Auto* mode

Paste these 5 parameters and raw data from Section L1-L4.opj projects

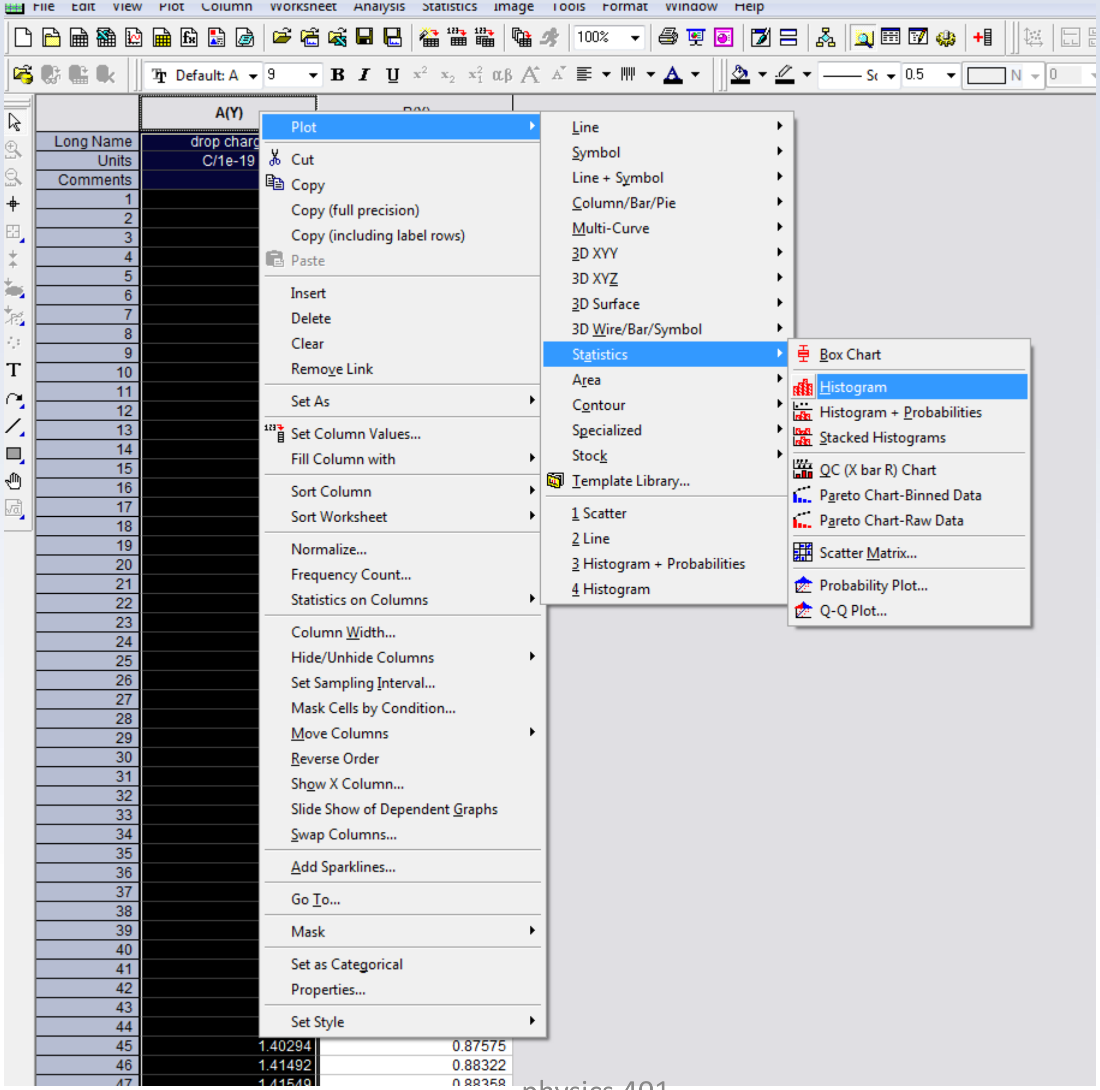
Calculate manually the actual air viscosity

Appendix. Analyzing of the statistical data.

Millikan oil drop experiment

Step 3. Histogram graph

First use the data from the column with drop charges and plot the histogram



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Appendix. Analyzing of the statistical data.

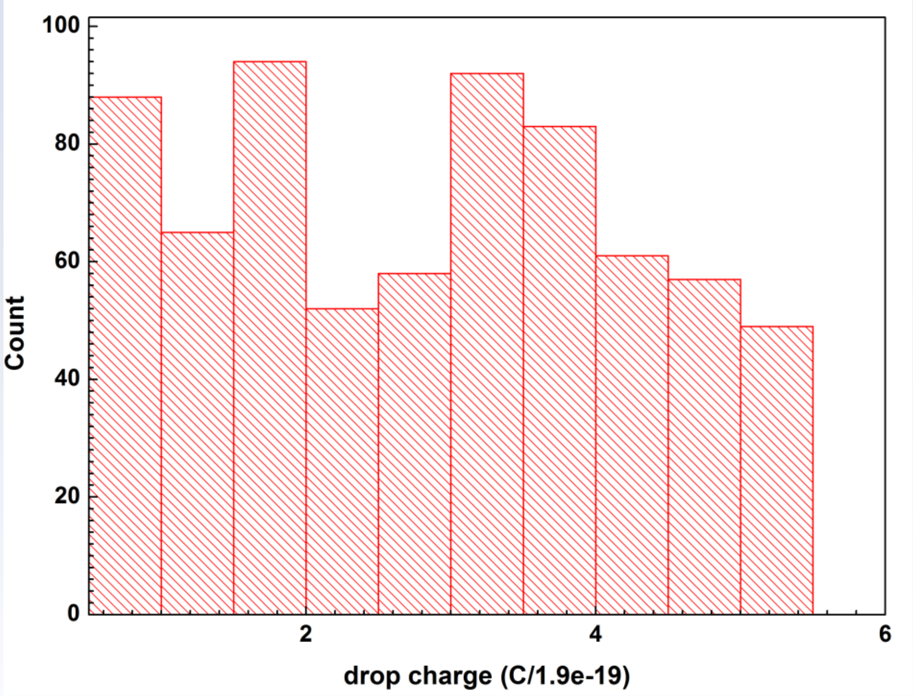
Millikan oil drop experiment

Step 4. Histogram. Bin size

Origin will automatically but not optimally adjust the bin size h. In tis page figure **h=0.5**. There are several theoretical approaches how to find the optimal bin size.

$$h = \frac{3.5\sigma}{n^{1/3}}$$

σ Is the sample standard deviation and **n** is total number of observation. For presented in Fig.1 results good value of **h ~0.1**

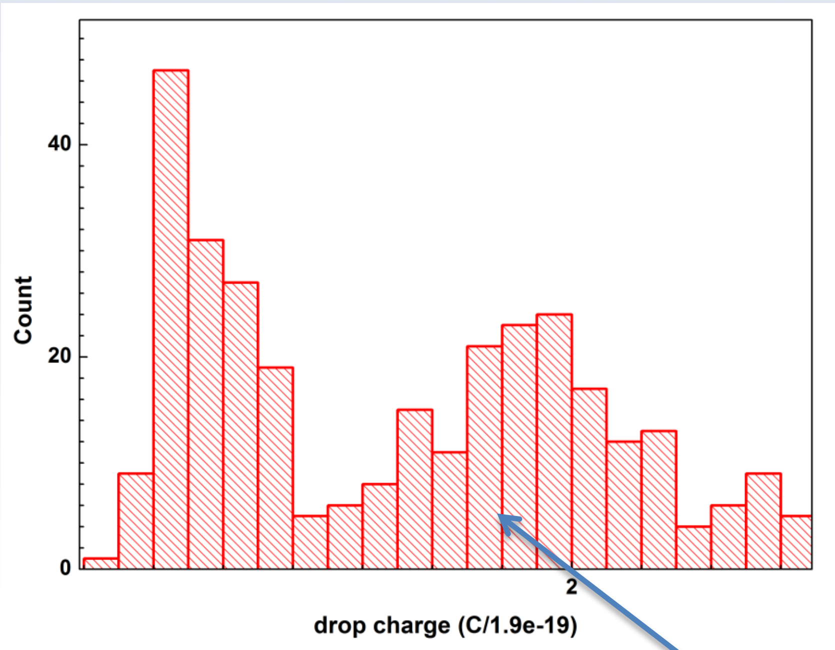


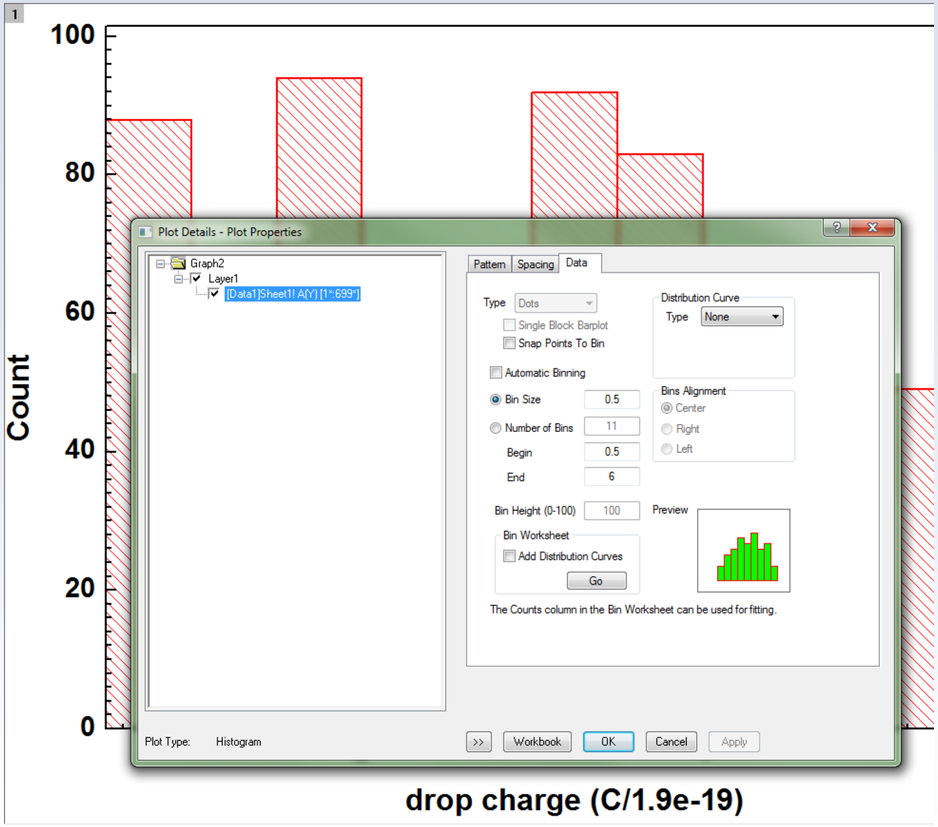
Appendix. Analyzing of the statistical data.

Millikan oil drop experiment

Step 4. Histogram. Bin size

To change the bin size click on graph and unplug the "***Automatic Binning***" option





Bin size in this histogram is 0.1

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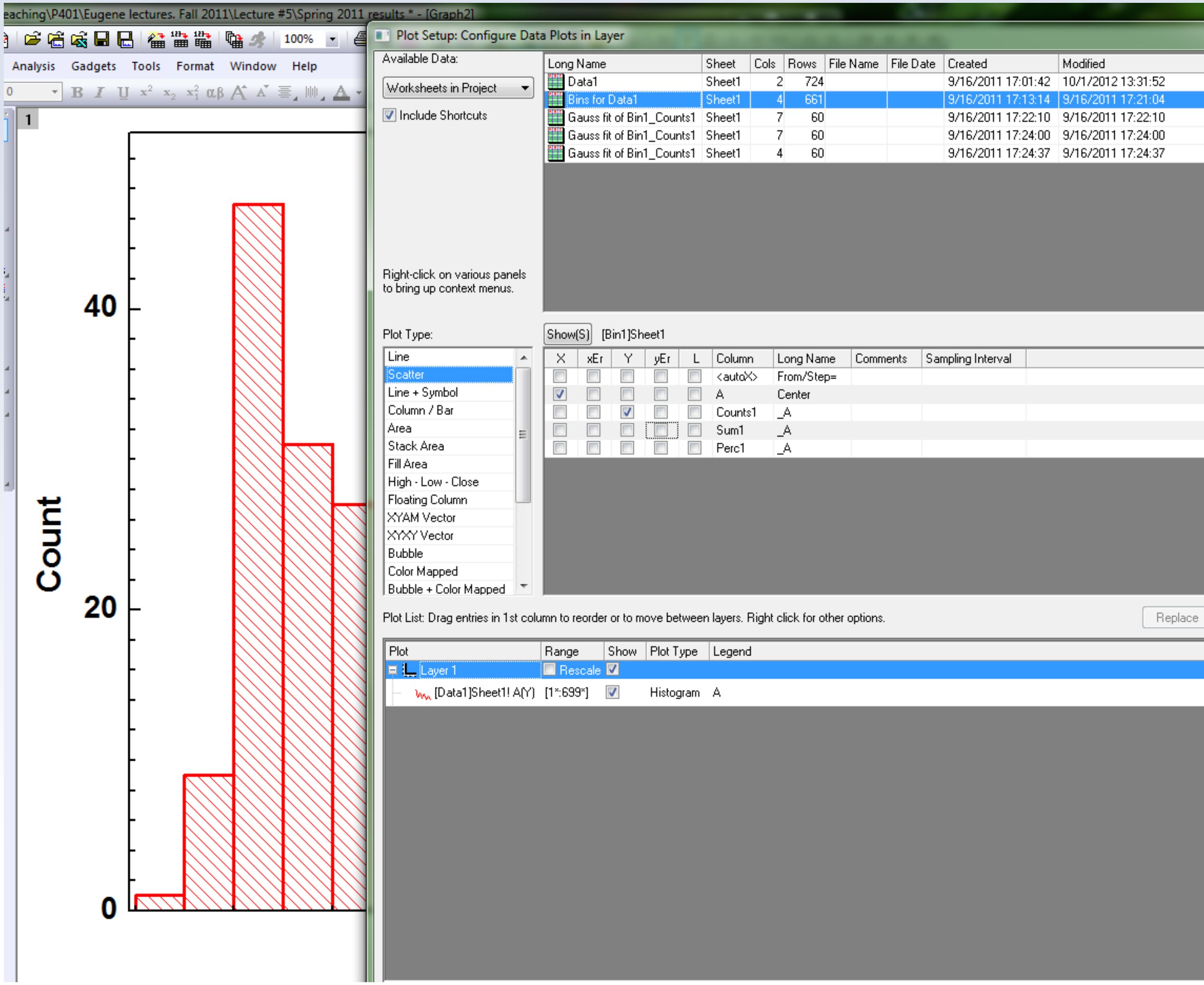
Appendix. Analyzing of the statistical data.

Step 4. Multipeak Gaussian fitting

Millikan oil drop experiment

To do this you have to add an extra plot to the graph

Counts vs. Bin Center

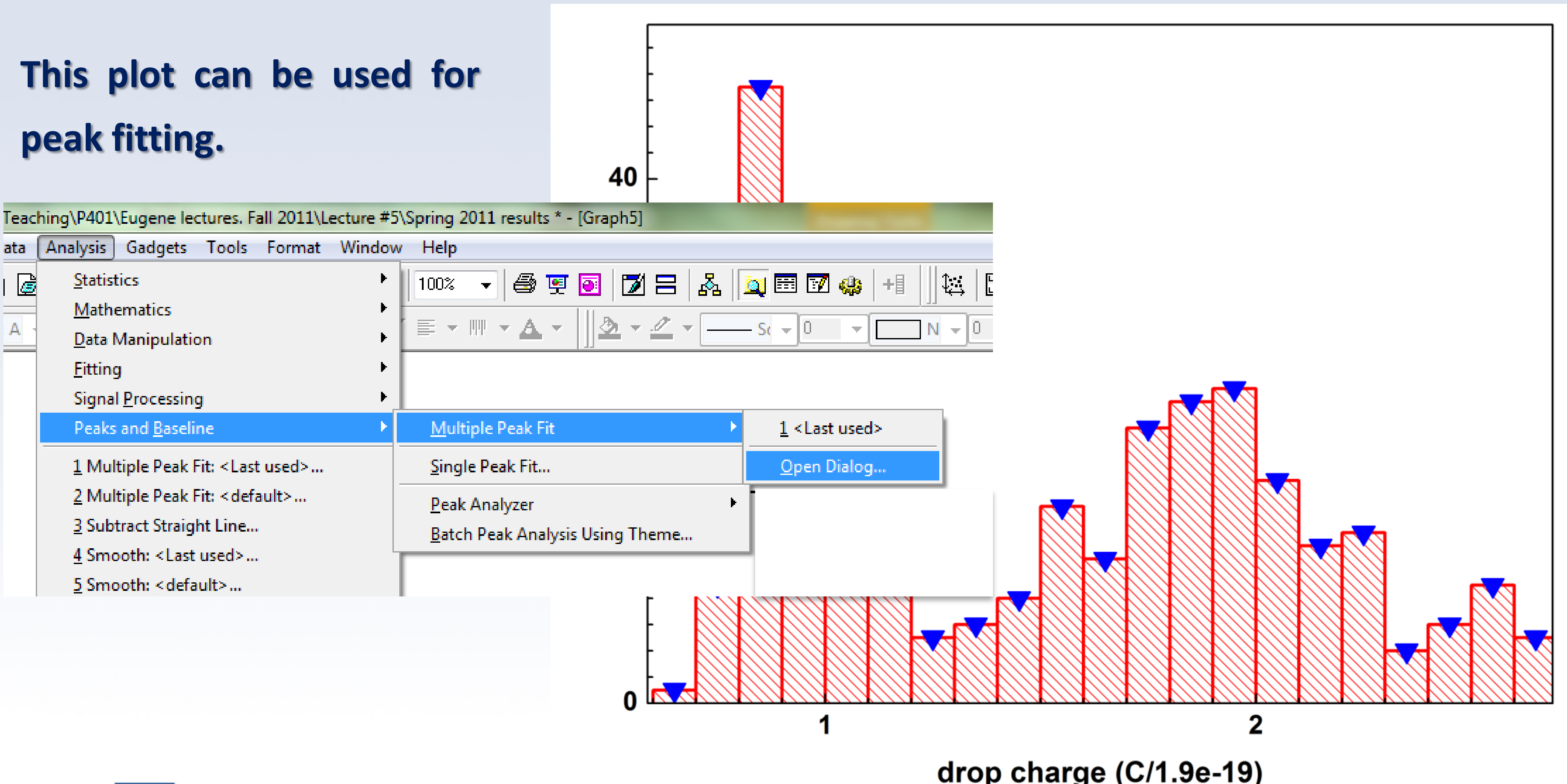


Appendix. Analyzing of the statistical data.

Millikan oil drop experiment

Step 4. Multipeak Gaussian fitting

This plot can be used for peak fitting.



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Appendix. Analyzing of the statistical data.

Millikan oil drop experiment

Step 4. Multipeak Gaussian fitting

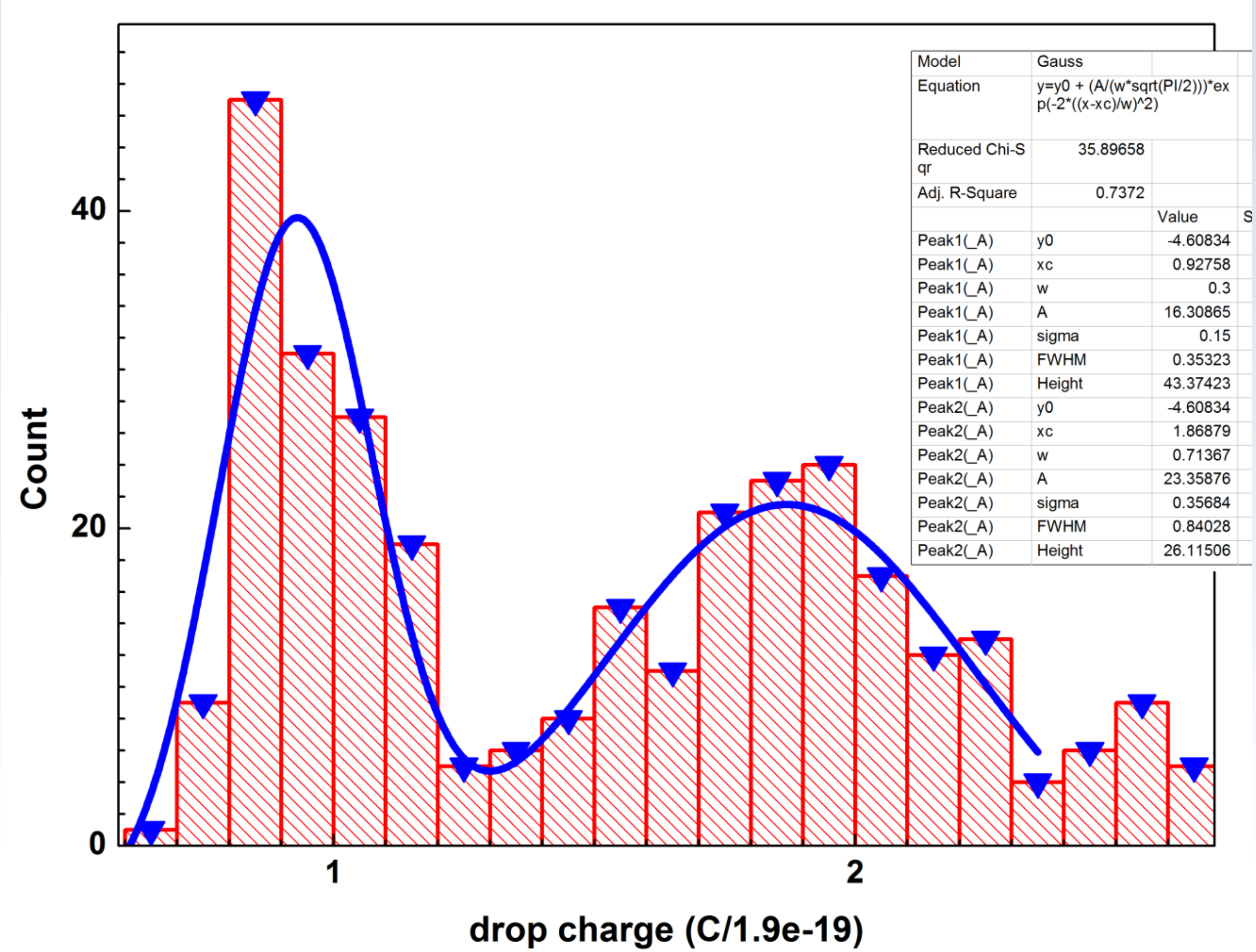
This plot can be used for peak fitting.

Final result for first two peaks:

$Q=0.93\pm0.013\times10^{-19}C$

$Q=1.87\pm0.024\times10^{-19}C$

This pretty close to e and 2e



Here $w = 2\sigma$ *and error of the mean* $= \frac{\sigma}{\sqrt{N}}$